Math 320 Linear Algebra
Assignment # 13

1. Consider
$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 4 \\ 0 & a & -1 \end{bmatrix}$$
Given that $\det(A) = \frac{2}{3}$, find A (i.e. find a the only missing part of A).

2. Consider:
$$C = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 4 & -8 & -9 & 1 \\ -6 & -2 & 7 & 10 \\ 0 & 8 & -7 & -5 \end{bmatrix}.$$
   (a) Show that $C = LU$ where:
$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & -2 & 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 0 & -4 & -1 & 3 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$
   This is called the LU-decomposition for C where we can write a matrix as a product of a lower triangular matrix with diagonals of 1 and an upper triangular matrix. The way to get it is by using a modified version of the Gaussian elimination.

   (b) Use this to find $\det(C)$.

3. Show that $(kA)^T = kA^T$.

4. Suppose that E is an elementary matrix.
   (a) Show that E^T is an elementary matrix of the same type and $\det(E^T) = \det(E)$.
   (b) Which type(s) of elementary matrices are orthogonal?

5. Suppose that A is orthogonal. Show that $\det(A)$ is either 1 or -1.

6. Let:
$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{bmatrix}.$$
Suppose $B = \frac{1}{3}A$.
   (a) Show that B is orthogonal.
   (b) Use this to find the possible values for $\det(A)$. (You don't need to find which one it is but you can for practice if you want).