Math 320 Linear Algebra
Assignment # 12

1. Consider:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & -2 & 5 \\ 0 & 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & -2 & 4 \\ 1 & 2 & 3 & 5 & -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 & 5 & -1 & 2 \\ 0 & 0 & 1 & -1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

   where B is the output of doing Gaussian elimination on A.

   (a) Find elemenry matrices $E_1, E_2, \ldots, E_r$ such that:

   $$E_r E_{r-1} \ldots E_2 E_1 A = B$$

   (b) Find an invertible matrix C such that $CA = B$.

   (c) Do the matrix multiplication to show that indeed $CA = B$.

   (d) Find $E_1^{-1}, E_2^{-1}, \ldots, E_r^{-1}$ (that is find the inverse of the elementary matrices found above).

   (e) Use the previous part to find C^{-1}.

   (f) Verify by matrix multiplication that $A = C^{-1}B$.

2. Consider the ordered basis:

$$\mathscr{B} = (\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix})$$

   for $\mathbb{R}^{2\times 2}$ and

$$\mathscr{D} = (x^3 + 1, x^2 + 1, x + 1, x^3)$$

   be an ordered basis for P_3.

   Also let $T_1 : \mathbb{R}^{2\times 2} \to P_3$ defined by:

$$T_1\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a + 2c)x^3 + ax^2 + (b + d)x + 2b + 2d.$$

   Also let: $T_2 : P_3 \to \mathbb{R}^{2\times 2}$ be defined by:

$$T_2(ax^3 + bx^2 + cx + d) = \begin{bmatrix} a & a + b \\ a + c & 2a + b \end{bmatrix}$$

   Find:

   (a) $\text{Rep}_{\mathscr{B},\mathscr{D}}(T_1)$

   (b) $\text{Rep}_{\mathscr{D},\mathscr{B}}(T_2)$

   (c) $\text{rank}(T_1)$

   (d) $\text{nullity}(T_2)$