

## Math 320 Linear Algebra Assignment # 6

1. For each of the vector spaces  $V$  and set of vectors  $\mathcal{A}$  below do the following: i) determine if the set of vectors  $\mathcal{A}$  is linearly independent in the vector space  $V$  (either find a non-trivial relation or show one does not exist). If the set is linearly dependent then also do: ii) remove as few vectors as possible from  $\mathcal{A}$  to create a new linearly independent set of vectors called  $\mathcal{A}'$  iii) show the new set,  $\mathcal{A}'$ , is linearly independent.

- (a)  $V = \mathbb{R}^{2 \times 2}$  and

$$\mathcal{A} = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right\}$$

- (b)  $V = P_4$  and  $\mathcal{A} = \{x^2 - x, x^2 + x, x^3, x^2 + x^3, 1\}$

- (c)  $V = \mathbb{R}^5$  and

$$\mathcal{A} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -18 \\ -36 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 6 \\ 12 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 9 \\ 18 \\ -15 \end{bmatrix} \right\}$$

2. Let  $V = \mathcal{F}(\mathbb{R}, \mathbb{R})$ . Determine with proof if the following are linearly independent.

- (a)  $\{\sin(x), \cos(x)\}$

(Hint: the  $\mathbf{0}$  in  $V$  is the zero function call it  $\mathbf{0}$ , that is the function  $g$  so that  $g(x) = 0$  for all  $x \in \mathbb{R}$ . So suppose that  $a \sin(x) + b \cos(x)$  is the zero function use appropriate values of  $x$  to show that  $a = b = 0$ .)

- (b)  $\{e^x, x\}$

- (c)  $\{\sin(x), e^x, \sin^2(x), \cos^2(x), 3\}$  (Note: 3 is the constant function 3.)