Math 320 Linear Algebra
Assignment # 11

1. Consider :

$$A = \begin{bmatrix} 1 & -2 \\ 1 & -3 \\ 4 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 & y \\ x & 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 10 & -3 & 2 \\ 16 & -5 & 1 \\ -2 & 2 & z \end{bmatrix}$$

   (a) Find x, y, and z so that $AB = C$.

   (b) Find BA.

2. Consider:

$$A^{-1} = \begin{bmatrix} 3 & 2 & -1 \\ 1/2 & 0 & 4 \\ -2 & 1 & -2 \end{bmatrix}, B^{-1} = \begin{bmatrix} 3/2 & -2 & 1 \\ 3 & 0 & -4 \\ 0 & 1 & -2 \end{bmatrix}$$

   Suppose $C = AB$. Find C^{-1}.

3. In the last homework you showed that $T_1 : W \to V$ and $T_2 : V \to U$ are linear transformation then:

$$\mathscr{R}(T_2 \circ T_1) \leq \mathscr{R}(T_2)$$
$$\mathscr{N}(T_1) \leq \mathscr{N}(T_2 \circ T_1)$$

   You may use those results in the following. Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times p}$.

   (a) Show that $\text{rank}(AB) \leq \text{rank}(A)$

   (b) Show that $\text{rank}(AB) \leq \text{rank}(B)$

   (c) Show that $\text{rank}(AB) \leq \min(m, n, p)$. (Note $\min(m, n, p)$ is the smallest of the three numbers m, n and p. That is show $\text{rank}(A) \leq m, \text{rank}(A) \leq n$, and $\text{rank}(A) \leq p$.)

   (d) Suppose $\vec{w}, \vec{v}$ are non-zero elements of $\mathbb{R}^n = \mathbb{R}^{n\times 1}$. Let $A = \vec{w}\vec{v}^T$.

      i. What is $\text{rank}(A)$.

      ii. Let $\vec{w} = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 0 \\ 12 \\ 1 \end{bmatrix}$. Calculate A in this case.

4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and define $\det(A) = ad - bc$.

   (a) Show that if $\det(A) = 0$ then A is singular. (Hint use two cases, $a = 0$ and $a \neq 0$.)

   (b) Conversely show that if $\det(A) \neq 0$ then A is invertible and $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.