Math 320 Linear Algebra
Assignment # 14

1. Consider

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 4 \\ 0 & a & -1 \end{bmatrix}$$

Given that $\det(A) = \frac{2}{3}$, find A (i.e. find a the only missing part of A).

2. Consider:

$$C = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 4 & -8 & -9 & 1 \\ -6 & -2 & 7 & 10 \\ 0 & 8 & -7 & -5 \end{bmatrix}.$$

   (a) Show that $C = LU$ where:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & -2 & 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 0 & -4 & -1 & 3 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

   This is called the LU-decomposition for C where we can write a matrix as a product of a lower triangular matrix with diagonals of 1 and an upper triangular matrix. The way to get it is by using a modified version of the Gaussian elimination.

   (b) Use this to find $\det(C)$.

3. Let:

$$A = \begin{bmatrix} 2 & 5 & 0 & 3 \\ -1 & a & 2 & 5 \\ 0 & 2 & 0 & -1 \\ -2 & -1 & 0 & 0 \end{bmatrix}$$

   (a) Find $\det(A)$

   (b) Notice that $\det(A)$ does not depend on a. Are there any other values that can be change (while not changing any other values) that does not change the determinate?

4. Suppose that $\lambda \in \mathbb{R}$ is an eigenvalue for $A \in \mathbb{R}^{n \times n}$. (That is there exists $\vec{v_0} \neq 0$ called an eigenvector such that $A\vec{v_0} = \lambda \vec{v_0}$). Let $E_\lambda = \{\vec{v} \in \mathbb{R}^n : A\vec{v} = \lambda v\}$. Show that E_λ is a subspace of $\mathbb{R}^n$.

5. Let

$$A = \begin{bmatrix} 14/3 & -5/3 & 1 \\ 17/3 & -8/3 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$$

Consider:

$$\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} \right\}.$$

   (a) Show that $\mathscr{B}$ is an eigenbasis with respect to A and find the corresponding eignenvalues?

   (b) Find D and P so that $A = PDP^{-1}$.

6. Find an eigenbasis for

$$A = \begin{bmatrix} -3 & -3 & 6 \\ 0 & 0 & -6 \\ 0 & 0 & -3 \end{bmatrix}$$

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