Math 320 Linear Algebra
Assignment # 9

1. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and define $\det(A) = ad - bc$.

   (a) Show that if $\det(A) = 0$ then A is does not row reduce to the identity matrix and hence is not invertible (i.e is singular). (Hint use two cases, $a = 0$ and $a \neq 0$.)

   (b) Conversely show that if $\det(A) \neq 0$ then A is invertible and $A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ by showing that $AA^{-1} = I_2$.

2. Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a 1-1 and onto linear transformation. Finish showing $T^{-1} : \mathbb{R}^n \to \mathbb{R}^n$ is also linear by showing that for all $c \in \mathbb{R}$ and $\vec{v} \in \mathbb{R}^n$, $T^{-1}(c\vec{v}) = cT^{-1}(\vec{v})$.