Math 320 Linear Algebra
Assignment # 11

1. Show that vector space properties 4,5 and 9 hold for the vector space P_n (that is the space of polynomials of degree n or less.)

2. The polynomials $x+1$ and x^2+2 are "vectors" in the vector space P_2. Describe the set span$(x+1, x^2+2)$.

3. Let V be a vector space.

   (a) Show that if $\vec{v}, \vec{w} \in V$ and $\vec{v}+\vec{w}=\vec{0}$ then $\vec{w}=-\vec{v}$.

   (b) Show that if $\vec{v} \in V$ then $(-1)\vec{v}=-\vec{v}$.

4. Let V be a vector space and $W=\{\vec{0}\}$. Show that W is a subspace of V. This is called the trivial subspace of V.

5. Let V be a vector space, and $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \in V$. Let $H=\text{span}(\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p)$. Show that H is a subspace of V.

   I made a short video that might help with this problem: Spanning set Proof video

6. For each of the following determine (with proof) if W is a subspace of the vector space V.

   (a) $V=\mathbb{R}^4$ and
   $$W=\left\{\begin{bmatrix} a+2b \\ 0 \\ 3a+b \\ c \end{bmatrix} : a,b,c \in \mathbb{R}\right\}$$

   (b) $V=P_4$ and
   $$W=\left\{ax^3+bx^2+2x+c : a,b,c \in \mathbb{R}\right\}$$

   (c) $V=\mathbb{R}^{2\times 2}$ and
   $$W=\left\{\begin{bmatrix} a & a^2 \\ b & 0 \end{bmatrix} : a,b \in \mathbb{R}\right\}$$

   (d) $V=\mathbb{R}^3$ and
   $$W=\left\{\vec{x} \in \mathbb{R}^3 : A\vec{x}=\vec{b}\right\}$$
   where:
   $$A=\begin{bmatrix} 1 & -2 & 7 \\ 3 & -2 & -1 \\ -1 & 8 & 2 \end{bmatrix}, \quad \vec{b}=\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

   (e) $V=\mathscr{F}(\mathbb{R},\mathbb{R})$ (the set of all function from the $\mathbb{R}$ to the $\mathbb{R}$).
   $$W=\{f \in \mathscr{F}(\mathbb{R},\mathbb{R}) : f(2)=0\}.$$