Math 320 Linear Algebra
Assignment # 11

1. Let $T : P_2 \to \mathbb{R}^{2\times 2}$ be defined by:

$$T(ax^2 + bx + c) = \begin{bmatrix} a+b & a \\ b & c \end{bmatrix}$$

   You can assume without proof that T is linear. (Hint: Only the first problem should require much in the way of work, the rest can be solved from theorems in the class.)

   (a) Find a basis for $\ker(T)$.

   (b) Find $\dim(\ker(T))$.

   (c) Find $\dim(\text{Rg}(T))$

   (d) Find a basis for $\text{Rg}(T)$

   (e) Is T 1-1?

   (f) Is T onto?

2. Let V and W be vector spaces with $\dim(V) = n$ and $\dim(W) = m$. Also $T : V \to W$ is a linear transformation. Determine which of the following statements are **always** true explain your answer.

   (a) If $m < n$ then T is not 1-1.

   (b) If $m > n$ then T is not 1-1.

   (c) If $m < n$ then T is not onto.

   (d) If $m > n$ then T is not onto.

3. Consider

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 4 \\ 0 & a & -1 \end{bmatrix}$$

   Given that $\det(A) = \frac{2}{3}$, find A (i.e. find a the only missing part of A).

4. Consider:

$$C = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 4 & -8 & -9 & 1 \\ -6 & -2 & 7 & 10 \\ 0 & 8 & -7 & -5 \end{bmatrix}.$$

   (a) Show that $C = LU$ where:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 0 & -2 & 3 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} -2 & 2 & 4 & 1 \\ 0 & -4 & -1 & 3 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}.$$

   This is called the LU-decomposition for C where we can write a matrix as a product of a lower triangular matrix with diagonals of 1 and an upper triangular matrix. The way to get it is by using a modified version of the Gaussian elimination.

   (b) Use this to find $\det(C)$.

5. Suppose that $\lambda \in \mathbb{R}$ is an eigenvalue for $A \in \mathbb{R}^{n\times n}$. (That is there exists $\vec{v_0} \neq 0$ called an eigenvector such that $A\vec{v_0} = \lambda\vec{v_0}$). Let $E_\lambda = \{\vec{v} \in \mathbb{R}^n : A\vec{v} = \lambda v\}$. Show that E_λ is a subspace of $\mathbb{R}^n$.