Math 320 Linear Algebra
Assignment # 4

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by:

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+b \\ 3c \end{bmatrix}$$

   (a) Find:

$$5 \cdot T\left(\begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}\right)$$

   (b) Find:

$$T\left(5 \cdot \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}\right)$$

   (c) Are the previous two parts equal? Does that guarantee that T is linear?

   (d) Prove that T is linear by using the one step verification.

2. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by:

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a-2b \\ 3b \\ 2a-b \end{bmatrix}$$

   (a) Either prove that T is linear by finding A such that $T(\vec{v}) = A\vec{v}$, or show that T is not linear with a specific example.

   (b) Find $\vec{v}$ such that $T(\vec{v}) = \left[\begin{smallmatrix} 10 \\ -9 \\ 11 \end{smallmatrix}\right]$, or explain why no such $\vec{v}$ exists.

   (c) Find $\vec{v}$ such that $T(\vec{v}) = \left[\begin{smallmatrix} 1 \\ 2 \\ -1 \end{smallmatrix}\right]$, or explain why no such $\vec{v}$ exists.

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by:

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} a \\ a^2+1 \\ -b \end{bmatrix}$$

   (a) Either prove that T is linear by finding A such that $T(\vec{v}) = A\vec{v}$, or show that T is not linear with a specific example.

   (b) Find $\vec{v}$ such that $T(\vec{v}) = \left[\begin{smallmatrix} 10 \\ -9 \\ 11 \end{smallmatrix}\right]$, or explain why no such $\vec{v}$ exists.

   (c) Find $\vec{v}$ such that $T(\vec{v}) = \left[\begin{smallmatrix} 1 \\ 2 \\ -1 \end{smallmatrix}\right]$, or explain why no such $\vec{v}$ exists.

4. Suppose that $T : \mathbb{R}^n \to \mathbb{R}^m$ has the one-step verification property, that is for all $\vec{u}, \vec{v} \in \mathbb{R}^n$ and $c, d \in \mathbb{R}$, $T(c\vec{u} + d\vec{v}) = cT(\vec{u}) + dT(\vec{v})$. Show that $T(c\vec{u}) = cT(\vec{u})$. This finishes the proof of the one-step verification.