Math 320 Linear Algebra
Assignment # 9

1. Let V be a vector space and $W = \{\vec{0}\}$. Show that W is a subspace of V. This is called the trivial subspace of V.

2. Let V be a vector space, and $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p \in V$. Let $H = \text{span}(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_p)$. Show that H is a subspace of V.

   I made a short video that might help with this problem: Spanning set Proof video

3. Let V, W be vector spaces and $T : V \to W$ be a linear transformation. Show that $\ker(T)$ is a subspace (not just a subset) of V.

   In the following video I show it is true for $\text{Rg}(T)$. Range of T is a subspace video

4. For each of the following determine (with proof) if W is a subspace of the vector space V.

   (a) $V = \mathbb{R}^4$ and
   $$W = \left\{ \begin{bmatrix} a+2b \\ 0 \\ 3a+b \\ c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$$

   (b) $V = P_4$ and
   $$W = \left\{ ax^3 + bx^2 + 2x + c : a, b, c \in \mathbb{R} \right\}$$

   (c) $V = \mathbb{R}^{2\times 2}$ and
   $$W = \left\{ \begin{bmatrix} a & a^2 \\ b & 0 \end{bmatrix} : a, b \in \mathbb{R} \right\}$$

   (d) $V = \mathbb{R}^3$ and
   $$W = \left\{ \vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{b} \right\}$$
   where:
   $$A = \begin{bmatrix} 1 & -2 & 7 \\ 3 & -2 & -1 \\ -1 & 8 & 2 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$$

   (e) $V = \mathscr{F}(\mathbb{R}, \mathbb{R})$ (the set of all function from the $\mathbb{R}$ to the $\mathbb{R}$).
   $$W = \{ f \in \mathscr{F}(\mathbb{R}, \mathbb{R}) : f(2) = 0 \}.$$

5. For each of the following either show the transformation is a linear transformation or show it is not. Here are some videos that could help:
   Proving a function is a linear transformation
   Proving a function is not a linear transformation

   (a) $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ defined by:
   $$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = \begin{bmatrix} 2a+b \\ a \\ 0 \end{bmatrix}.$$

(b) Let $T : P_2 \to \mathbb{R}^3$ defined by:

$$T(ax^2 + bx + c) = \begin{bmatrix} a+b \\ ac \\ a \end{bmatrix}$$

6. Each of the following you may assume are linear transformation. For each find a basis for both $\text{Rg}(T)$ and $\ker(T)$. If you need another example of how to find a basis here is a video: Find a basis for Kernel and Range of a Transformation

(a) $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^3$ defined by:

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b \\ a-b \\ c \end{bmatrix}.$$

(b) Let $T : P_2 \to \mathbb{R}^3$ defined by:

$$T(ax^2 + bx + c) = \begin{bmatrix} a+b \\ a+c \\ a \end{bmatrix}$$

(c) $T : \mathbb{R}^4 \to \mathbb{R}^2$ defined by $T(\vec{v}) = A\vec{v}$ where:

$$A = \begin{bmatrix} 3 & 3 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$$

(d) $T : P_2 \to \mathbb{R}$ defined by $T(p(x)) = \int_0^1 p(x)$.