

Suppose $S = \{1, 2, 3, 4\}$ and

$$P(\{1\}) = \frac{\sqrt{2}}{2} - \frac{1}{4}$$

$$P(\{2\}) = \frac{1}{4}$$

$$P(\{3\}) = -\frac{\sqrt{2}}{2} + \frac{3}{4}$$

$$P(\{4\}) = \frac{1}{4}$$

$$E_1 = \{1, 3\}$$

$$E_2 = \{2, 3\}$$

$$E_3 = \{3, 4\}.$$

Show that: $P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3)$ but that no pair of events E_1, E_2 , and E_3 are independent and hence E_1, E_2 , and E_3 are not independent.