

## Math 350 Probability – Exam 1 – Fall 2007

Instructions: **Answer each question completely and show all work.**

1. You are dealt 3 cards from a standard deck. What is the probability that you either have at least two aces or at least two kings?
2. A fair  $k$ -sided die is one in which the numbers  $\{1, 2, \dots, k\}$  are rolled with equal probability. Suppose fair 4, 6, 8, 12 and 20-sided dice are placed in a bag and selected at random (each of the 5 dice is equally likely to be selected) and then that die is rolled.
  - (a) What is the probability a 7 is rolled?
  - (b) What is the conditional probability that the 8-sided die was selected given that a 7 is rolled?
  - (c) Suppose that a die selected from the bag and rolled 3 times (so the same die is rolled 3 times). What is the probability that one of the 4-sided dice was chosen given that all 3 rolls are between 1 and 4?
3. In a certain community, 36% of the families own a dog, and 22% of the families that own a dog also own a cat. In addition, 30% of families own a cat.

What is:

  - (a) the probability that a randomly selected family owns both a dog and a cat
  - (b) the probability that a randomly selected family owns a dog given that it owns a cat?
4. Suppose that events  $E_1$ ,  $E_2$  and  $E_3$  are independent with  $P(E_1) = p_1$ ,  $P(E_2) = p_2$  and  $P(E_3) = p_3$ . Further suppose that  $E_4$  is disjoint with each of  $E_1, E_2, E_3$  and  $P(E_4) = p_4$ .
  - (a) Find the probability of that:
    - i. both  $E_1$  and  $E_2$  occur but not  $E_3$ .
    - ii. at least one of the events  $E_1, E_2$  or  $E_3$  occurs.
    - iii. at least one of the events  $E_1, E_2, E_3$  or  $E_4$  occurs.
    - iv. exactly two of the four events occur.
  - (b) Give an example of an experiment and four events  $E_1, E_2, E_3$  and  $E_4$  that satisfy the above conditions.
5. Prove that if  $A, B, C, D$  are independent events then the events  $A \cap B$  and  $C \cap D$  are independent.

**You may turn a solutions to this question on Monday, but you cannot talk to other people about it.**

6. Bonus Question Let  $A_1, A_2, \dots$  be an infinite sequence of independent events with:

$$P(A_k) = \frac{1}{2}$$

Prove that:

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = 1.$$

(Hint show that:

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) > 1 - \epsilon$$

for any  $\epsilon > 0$ .