Math 350 Probability – Exam 2 – Fall 2007

Instructions:**Answer each question completely and show all work.**

1. Suppose an urn contains 5 red balls and 3 black balls. Also suppose that balls are withdrawn from the urn one at a time *without* replacement. Let X be the number of balls withdrawn before a red ball is drawn (including the draw that contains the red ball).

   (a) Find the probability mass function $p_X(k)$ for X.

   (b) Find $\mathrm{E}(X)$.

   (c) Find $\mathrm{Var}(X)$.

2. Suppose $X \sim \mathcal{B}(1000, \frac{1}{50})$. Estimate $P(17 \leq X \leq 21)$ using:

   (a) the Poisson Distribution

   (b) the normal Distribution

3. An airline finds that 7% of the people making reservations on a certain flight will not show up for the flight.

   If the airline sells 164 tickets for a flight with only 155 seats, what is the probability that a seat will be available for every person holding a reservation who actually arrives for the flight.

4. Let X be a continuous random variable with pdf given by:

$$f_X(x) = \begin{cases} \frac{c}{x^2} & \text{if } x \geq 1 \\ 0 & \text{if } x < 1. \end{cases}$$

   (a) What is the value of c?

   (b) Find the cumulative distribution function of X.

   (c) What can you say about $E(X)$?

5. You arrive at a bus stop at 10 o'clock, knowing that the bus will arrive at some time uniformly distributed between 10 and 10:30.

   (a) What is the probability that you will have to wait longer than 10 minutes?

   (b) If at 10:15 the bus has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

6. Suppose that an average of 7 calls come into a call center every minute. Find the distribution of the following random variables including the values of any parameters also find the expected value and variance of each random variable.

   (a) X where X is the number of calls in a 5 minute period.

   (b) T_1 where T_1 is the amount of time until the first call.

   (c) T_2 where T_2 is the amount of time between the third and forth call.

   (d) Y where Y is the number of minutes amongst the first 1000 minutes in which there are no calls.

   (e) Find an approximate distribution of Y in part 6d.