

Problems from Assignment 13

1. Let  $X$  have probability density function given by:

$$f_X(x) = \begin{cases} 2(1-x), & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let  $W = X^2$ . It can be shown (and we will show it later) that the pdf of  $W$  is:

$$f_W(w) = \begin{cases} \frac{1}{\sqrt{w}} - 1, & 0 \leq w \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the  $E(W)$  in two ways one using the pdf of  $W$  and one using the pdf of  $X$ .

2. Let  $X$  be a random with pdf given by:

$$f_X(x) = \begin{cases} 4xe^{-2x}, & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show  $f_X(x)$  is indeed a pdf.
- (b) Find  $E(X)$ .
- (c) Show that the mode of  $X$  is  $\frac{1}{2}$ .
- (d) Find  $F_X(x)$ .
- (e) Show that the median of  $X$  is between 0.839 and 0.84.