Problems from Assignment 14

1. Let $X \sim \mathscr{G}(p)$ find:
   (a) $\mathrm{E}[(X+1)X]$ (look at how we computed $E(X)$ in class)
   (b) $\mathrm{E}(X^2)$
   (c) $\mathrm{Var}(X)$

2. We say that U has a uniform distribution on the interval $[a, b]$ (written $U \sim \mathscr{U}(a, b)$ with $a < b$) if is is continuous with the pdf:
$$f_U(u) = \begin{cases} \frac{1}{b-a}, & a \le u \le b \\ 0 & \text{otherwise.} \end{cases}$$
   (a) Show $f_U(u)$ is indeed a pdf.
   (b) Suppose $a \le c \le d \le b$, find $\mathrm{P}(c \le U \le d)$.
   (c) Graph $f_U(u)$ and guess what $E(U)$ should be.
   (d) Find $\mathrm{E}(U)$.
   (e) What about a and b do you think will make the variance bigger, smaller?
   (f) Find $\mathrm{Var}(U)$.
   (g) Find $F_U(u)$.
   (h) Find the median of U.