Problems from Assignment 18

1. (a) Suppose $X, Y, Z \overset{\text{iid}}{\sim} \mathscr{E}(\lambda)$ (remember we switched the parameterization of $\mathscr{E}$). Let $V = X + Y + Z$, find $f_V(v)$.

   Hint: Let $W = X + Y$ and remember we found $f_W(w)$ in class.

   (b) Suppose that X has pdf:

$$f_X(x) = \begin{cases} \frac{\lambda^k x^{k-1}}{(k-1)!} e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   Further suppose $Y \sim \mathscr{E}(\lambda)$ and is independent of X. Let $W = X + Y$ and show:

$$f_W(w) = \begin{cases} \frac{\lambda^{k+1} w^k}{k!} e^{-\lambda w} & w \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   (c) Suppose the $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathscr{E}(\lambda)$ and let $W = X_1 + X_2 + \ldots + X_n$. Use induction to show that:

$$f_W(w) = \begin{cases} \frac{\lambda^n w^{n-1}}{(n-1)!} e^{-\lambda w} & w \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   Hint: You have basically already proven this and you just need to notice you have proven it and put the pieces together.

2. Prove Theorem 3.8.5