Problems from Assignment 24

1. Let $X \sim \mathscr{E}(\lambda)$.
   (a) Show that:
   $$m_X(t) = \frac{\lambda}{\lambda - t}$$
   when $t < \lambda$.
   (b) Show that $m_X(0) = 1$.
   (c) Show that $m'_X(0) = \frac{1}{\lambda}$.
   (d) Show that $m''_X(0) = \frac{2}{\lambda^2}$.
2. Find $m_X(t)$ where X is defined as below:
   (a) If you last name starts with A-E then $X \sim \mathscr{G}(p)$
   (b) If you last name starts with F-O then $X \sim \mathscr{U}(a, b)$
   (c) If you last name starts with P-Z then X is discrete uniform on the set $\{1, 2, \ldots, n\}$. That is:
   $$f_X(k) = \begin{cases} \frac{1}{n} & k \in \{1, 2, 3, \ldots, n\} \\ 0 & \text{otherwise} \end{cases}$$
3. Show that if X has moment generating function and $W = aX + b$:
   $$m_W(t) = e^{bt} m_X(at)$$
4. Let $X \sim \mathscr{P}(\lambda)$ use moment generating functions to find:
   (a) $E(X)$
   (b) $E(X^2)$