

Problems from Assignment 10

1. Prove Theorem 3.8.5
2. Suppose $X, Y \stackrel{\text{iid}}{\sim} \mathcal{E}(\lambda)$ and $W = \frac{Y}{X}$ then in class we showed that:

$$f_W(w) = \begin{cases} \frac{1}{(w+1)^2} & w \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show that $f_W(w)$ is indeed a pdf.
 - (b) Find $P(X < Y)$. (You can do this problem without a calculation)
 - (c) Find the median of W .
 - (d) Find $E(W)$.
3. Let $X_1, X_2, X_3 \sim \mathcal{Ber}(\frac{1}{2})$ and let $W = (2X_1 - 1)(X_2 + X_3)$ and $V = W^2$.
 - (a) Find the pdf of W .
 - (b) Find $E(W)$
 - (c) Find $E(V)$
 - (d) Find $E(W^3)$
 - (e) Show that W and V are uncorrelated.
 - (f) Show that W and V are not independent.
 4. Suppose 8 people separately enter an elevator with eleven floors (not including the ground floor). Suppose that each of people independently choose a floor that is equally likely to be any of the 11. What is the expected number of floors that will be visited?