Problems from Assignment 7

1. Let X have probability density function given by:

$$f_X(x) = \begin{cases} 2(1-x), & 0 \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Let $W = X^2$. It can be shown (and we will show it later) that the pdf of W is:

$$f_W(w) = \begin{cases} \frac{1}{\sqrt{w}} - 1, & 0 \le w \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the $E(W)$ in two ways one using the pdf of W and one using the pdf of X.

2. Let X be a random with pdf given by:

$$f_X(x) = \begin{cases} 4xe^{-2x}, & x \ge 0 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show $f_X(x)$ is indeed a pdf.

   (b) Find $E(X)$.

   (c) Show that the mode of X is $\frac{1}{2}$.

   (d) Find $F_X(x)$.

   (e) Show that the median of X is between 0.839 and 0.84.

3. Let $X \sim \mathscr{G}(p)$ find:

   (a) $\mathrm{E}[(X+1)X]$ (look at how we computed $E(X)$ in class)

   (b) $\mathrm{E}(X^2)$

   (c) $\mathrm{Var}(X)$

4. We say that U has a uniform distribution on the interval $[a, b]$ (written $U \sim \mathscr{U}(a, b)$ with $a < b$) if is is continuous with the pdf:

$$f_U(u) = \begin{cases} \frac{1}{b-a}, & a \le u \le b \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Show $f_U(u)$ is indeed a pdf.

   (b) Suppose $a \le c \le d \le b$, find $\mathrm{P}(c \le U \le d)$.

   (c) Graph $f_U(u)$ and guess what $E(U)$ should be.

   (d) Find $\mathrm{E}(U)$.

   (e) What about a and b do you think will make the variance bigger, smaller?

   (f) Find $\mathrm{Var}(U)$.

   (g) Find $F_U(u)$.

   (h) Find the median of U.