Problems from Assignment 6

1. Suppose that $X \sim \mathscr{E}(\lambda)$ with $(\lambda > 0)$. Thus X has pdf:

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

   (a) Show $\displaystyle\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

   (b) Find $F_X(x)$ the cdf for X.

   (c) Show that X has the memory-less property, where if $s, t > 0$ then:

$$\mathrm{P}(X > s + t | X > s) = \mathrm{P}(X > t)$$

2. Let $X \sim \mathscr{G}(p)$ with $(0 < p \leq 1)$.

   (a) Let t be a positive integer find $F_X(t)$.

   (b) Let t be a positive integer find $\mathrm{P}(X > t)$.

   (c) Show that the geometric distribution has the discrete version of the memory-less property, where if s, t are positive integers then:

$$\mathrm{P}(X > s + t | X > s) = \mathrm{P}(X > t)$$

   (d) Explain why this makes sense with the interpretation of the geometric distribution.