Problems from Assignment 9

1. Suppose $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathscr{E}(\mu)$. What is $f_{X_1,X_2,\ldots,X_n}(x_1, x_2, \ldots, x_n)$.

2. (a) Suppose $X_1, X_2, X_3 \overset{\text{iid}}{\sim} \mathscr{E}(\lambda)$. Let $V = X_1 + X_2 + X_3$, find $f_V(v)$.
   Hint: Let $W = X + X_2$ and remember we found $f_W(w)$ in class.

   (b) Suppose $X_1, X_2, X_3, X_4 \overset{\text{iid}}{\sim} \mathscr{E}(\lambda)$. Let $V = X_1 + X_2 + X_3 + X_4$, find $f_V(v)$.

   (c) Suppose that $X$ has pdf:

$$f_X(x) = \begin{cases} \frac{\lambda^k x^{k-1}}{(k-1)!} e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   Further suppose $Y \sim \mathscr{E}(\lambda)$ and is independent of $X$. Let $W = X + Y$ and show:

$$f_W(w) = \begin{cases} \frac{\lambda^{k+1} w^k}{k!} e^{-\lambda w} & w \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   Note: What you have done is essentially prove (for those of you that know induction you can fill in the details) that if $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathscr{E}(\lambda)$ and let $W = X_1 + X_2 + \ldots + X_n$ then:

$$f_W(w) = \begin{cases} \frac{\lambda^n w^{n-1}}{(n-1)!} e^{-\lambda w} & w \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$