Problems from Assignment 13

1. Show that $\Gamma(x+1) = x\Gamma(x)$.
2. Find $\Gamma(\frac{9}{2})$.
3. Find:
$$\frac{\Gamma(\frac{17}{3})}{\Gamma(\frac{2}{3})}$$
4. Suppose $X \sim \Gamma(r, \lambda)$. We showed that:
$$m_X(t) = \left(\frac{\lambda}{\lambda - t}\right)^r = \left(1 - \frac{t}{\lambda}\right)^{-r}.$$
Use this to find:
   (a) $\mathrm{E}(X^3)$
   (b) $\mathrm{Var}(X^2)$
5. Suppose $X_1, X_2, X_3, \ldots \overset{\text{iid}}{\sim} \mathscr{P}(3)$. As is common practice let $\overline{X}_n = \frac{X_1+X_2+X_3+\ldots+X_n}{n}$.
   (a) Find the exact value $\mathrm{P}(2.6 \leq \overline{X}_2 \leq 3.1)$
   (b) Estimate $\mathrm{P}(2.6 \leq \overline{X}_{50} \leq 3.1)$