Problems from Assignment 12

1. Using the same level of rigor that we used in class show that for large n:

$$\ln\left(\frac{np}{x}\right)^x \approx -t\sqrt{npq} - \frac{1}{2}t^2q + O\left(\frac{1}{\sqrt{n}}\right).$$

   (Remember that $t = \dfrac{x-np}{\sqrt{npq}}$.)

2. Suppose that there is a major flood every two years.
   (a) Find the probability that in a given year there are five major floods.
   (b) Find the probability that there is 5 floods in a year in one of the next 100 years.

3. Suppose $Z \sim \mathrm{N}(0,1)$.
   (a) Find $P(-2 \le Z < -1)$.
   (b) Find (to the best you can) a such that $\mathrm{P}(-1 \le X \le a) = 0.6$.

4. Suppose $X \sim \mathrm{N}(-2,5)$.
   (a) Find $P(-2 \le X < -1)$.
   (b) Find $\mathrm{P}(X > 2)$.

5. Let $X \sim \mathscr{E}(\lambda)$.
   (a) Show that:

$$m_X(t) = \frac{\lambda}{\lambda - t}$$

   when $t < \lambda$.
   (b) Show that $m_X(0) = 1$.
   (c) Show that $m'_X(0) = \frac{1}{\lambda}$.
   (d) Show that $m''_X(0) = \frac{2}{\lambda^2}$.

6. Find $m_X(t)$ where X is defined as below:
   (a) If you last name starts with A-E then $X \sim \mathscr{G}(p)$
   (b) If you last name starts with F-O then $X \sim \mathscr{U}(a,b)$
   (c) If you last name starts with P-Z then X is discrete uniform on the set $\{1,2,\ldots,n\}$. That is:

$$f_X(k) = \begin{cases} \frac{1}{n} & k \in \{1,2,3,\ldots,n\} \\ 0 & \text{otherwise} \end{cases}$$

7. Show that if X has moment generating function and $W = aX + b$:

$$m_W(t) = e^{bt} m_X(at)$$

8. Let $X \sim \mathscr{B}(n,p)$ use moment generating functions to find:
   (a) $E(X)$
   (b) $E(X^2)$