Problems from Assignment 11

1. Suppose the $X \sim \mathscr{P}(\lambda)$. You may use that $\mathrm{E}(X) = \lambda$. Find:
   (a) $\mathrm{E}\left(X(X-1)\right)$
   (b) $\mathrm{E}(X^2)$
   (c) $\mathrm{Var}(X)$.

2. Suppose the number of calls to a call center each minute is a Poisson Distribution with mean 3. What is the probability there will be exactly 4 calls in a given minute given there is at least 1?

3. Suppose that N is a Poisson process with rate λ. Let X be the time until the second event occurs.
   (a) Using the same approach that we did with the waiting time for the first event, show that:

$$f_X(x) = \begin{cases} \lambda^2 x e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

   (b) Notice this the same pdf we got from the sum of two independent exponential distributions. Explain why that makes sense.
   (c) Suppose $\lambda = \frac{1}{4}$. Find $\mathrm{P}(3 \leq X \leq 9)$, using:
      i. $f_X(x)$ above
      ii. the distributions of N (remember N consists of infinitely many random variables) .