Suppose $S = \{1, 2, 3, 4\}$ and

$$
P(\{1\}) = \frac{\sqrt{2}}{2} - \frac{1}{4} \\
P(\{2\}) = \frac{1}{4} \\
P(\{3\}) = -\frac{\sqrt{2}}{2} + \frac{3}{4} \\
P(\{4\}) = \frac{1}{4}
$$

$E_1 = \{1, 3\}$

$E_2 = \{2, 3\}$

$E_3 = \{3, 4\}$.

Show that: $P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3)$ but that no pair of events $E_1, E_2, \text{ and } E_3$ are independent and hence $E_1, E_2, \text{ and } E_3$ are not independent.