

1. Let X and Y be random variables with joint density given by:

$$f_{X,Y}(x,y) = \begin{cases} c(y^2 - x^2)e^{-y} & \text{if } -y \leq x \leq y \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find c .
- (b) Find the marginal density of X .
- (c) Find the marginal density of Y .
- (d) Are X and Y independent?
- (e) Find $E[X]$.

Hint: You may use the fact that:

$$\int_0^{\infty} x^n e^{-x} dx = n!$$