

Problems from Assignment 19

1. Consider the setup from class. Suppose for that $\sum_{i=1}^t p_i = 1$ and $W_1, W_2, \dots, W_t \stackrel{\text{iid}}{\sim} N(0, 1)$. Then let:

$$\vec{p} = \begin{bmatrix} \sqrt{p_1} \\ \sqrt{p_2} \\ \vdots \\ \sqrt{p_t} \end{bmatrix}, \vec{W} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_t \end{bmatrix}.$$

Finally let:

$$\vec{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_t \end{bmatrix} = \vec{W} - (\vec{W} \cdot \vec{p})\vec{p}$$

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Find:

- (a) $\text{Var}(Q_i)$
- (b) $\text{Cov}(Q_i, Q_j)$ for $i \neq j$. (I know that we did this at the end of class but it was fast so it is worth it write it all clearly).