Problems from Assignment 17

1. In the proof from class we had to construct a orthogonal matrix starting with the top row being $\vec{p}$. Do this for one particular example. Let

$$\vec{p} = \begin{bmatrix} \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \\ \sqrt{\frac{1}{3}} \end{bmatrix}.$$

Find $\vec{v}_1, \vec{v}_2$ so that:

$$A = \begin{bmatrix} \vec{p}^T \\ \vec{v}_1^T \\ \vec{v}_2^T \end{bmatrix}.$$

is orthogonal.

2. Consider the setup from class. Suppose for that $\sum_{i=1}^{t} p_i = 1$ and $W_1, W_2, \ldots, W_t \overset{\text{iid}}{\sim} N(0,1)$. Then let:

$$\vec{p} = \begin{bmatrix} \sqrt{p_1} \\ \sqrt{p_2} \\ \vdots \\ \sqrt{p_t} \end{bmatrix}, \vec{W} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_t \end{bmatrix}.$$

Finally let:

$$\vec{Q} = \begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_t \end{bmatrix} = \vec{W} - (\vec{W} \cdot \vec{p})\vec{p}$$

.

Find:

(a) $\text{Var}(Q_i)$

(b) $\text{Cov}(Q_i, Q_j)$ for $i \neq j$. (I know that we did this in class but it is worth it write it all clearly for yourself).