Problems from Assignment 9

1. Show that $\Gamma(x+1) = x\Gamma(x)$.

2. For this problem you may use the fact that the density for the normal distribution is a density. That is you may use the fact that if $\sigma > 0$ and $\mu \in \mathbb{R}$ then:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx = 1.$$

   You may also use the fact that the density is symmetric around μ.

   (a) Calculate:

$$\int_{0}^{\infty} e^{-x^2}\, dx$$

   (b) Find $\Gamma(\frac{1}{2})$. Hint: Do a substitution and then use the previous part.

   (c) Calculate $\Gamma\left(\frac{11}{2}\right)$. Hint: Don't do an integral.

3. Let $X \sim \chi^2_3$, i.e. X has a chi-squared distribution with 3 degrees of freedom.

   (a) Fill in the blanks: $X \sim \Gamma(\_\_, \_\_)$.

   (b) What is $f_X(x)$ (your answer should not have Γ in it)?

   (c) Use Simpson's rule with 6 intervals to compute $\mathrm{P}(X \leq 6)$.

   (d) How does this compare with Table A.3 in your book?