Problems from Assignment 5

1. For this problem you may use the folloing integrals when $a > 0$ and $n \in \mathbb{N}$:

$$\int_0^a (1 - \frac{x}{a})^{n-1}\, dx = \frac{a}{n}$$
$$\int_0^a x(1 - \frac{x}{a})^{n-1}\, dx = \frac{a^2}{n(n+1)}$$
$$\int_0^a x^2(1 - \frac{x}{a})^{n-1}\, dx = \frac{2a^3}{n(n+1)(n+2)}.$$

Suppose $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \mathscr{U}(0, \theta)$ and let $X_{\text{min}}$ be the minimum of the $n$ values.

(a) Find the pdf of $X_{\text{min}}$. (Hint: $F_{X_{\text{min}}}(x) = 1 - \text{P}(X_{\text{min}} > x)$.)

(b) Check that it is indeed a pdf.

(c) Find $\text{E}(X_{\text{min}})$.

(d) Choose $c$ so $\hat{\theta}_1 = cX_{\text{min}}$ is an unbiased estimator.

(e) Find $\text{Var}(X_{\text{min}})$.

(f) Compare thie to $\text{Var}(X_{\text{max}})$ that we did in class, and explain why this makes sense.

(g) Find $\text{Var}(\hat{\theta}_1)$.

(h) How does the efficency compare to $\hat{\theta} = \frac{n+1}{n} X_{\text{max}}$.