

Problems from Assignment 11

1. (a) Suppose  $X$  and  $Y$  are independent random variables and suppose the  $f_Y(y)$  is an even function. Show that  $f_{Y/X}(w)$  is an even function.  
(b) Show if  $T \sim t_n$  then  $f_T$  is an even function. You may not use the density of  $T$  since we computed that using the fact that  $f_T$  is symmetric around the origin which you are now proving.
2. Let  $T \sim t_5$ .
  - (a) Write  $f_T(t)$  without any  $\Gamma$  in it.
  - (b) Use Simpson's rule with  $n = 6$  intervals to estimate  $P(0 \leq T \leq 1.5)$ .
  - (c) Using the above estimation, estimate  $P(T \geq 1.5)$ .
  - (d) Is this consistent with Table A.2 in the book?
3. Suppose  $X, Y \stackrel{\text{iid}}{\sim} N(0, 1)$  and let  $W = \frac{Y^2}{X^2}$ .
  - (a) Find  $f_W(w)$  (it should not include any  $\Gamma$  functions).
  - (b) Argue what  $P(W \leq 1) = \frac{1}{2}$  without doing integration.
  - (c) By doing a clever substitution, integrate  $f_W(w)$  to show that  $P(W \leq 1) = \frac{1}{2}$ .