Math 351 Mathematical Statistics – Final Exam
Spring 2020

Name: ____________________

1. Suppose $X_1, X_2 \overset{\text{iid}}{\sim} N(0,1)$.
   (a) What is the distribution of $3X_1 - 2X_2$?
   (b) What is $\mathrm{Var}(3X_1X_2)$?
   (c) What is $\mathrm{E}(3X_1^2 - 2X_2^2)$?
   (d) What is $\mathrm{Var}(3X_1^2 - 2X_2^2)$?
   (e) Find $a, b, c \in \mathbb{R}$ so that $a(X_1 + X_2)$ and $bX_1 + cX_2$ are independent standard normal random variables.

2. One group of 9 men drank half a bottle of red wine each day for two weeks. The level of polyphenols in their blood was measured before and after the two-week period. Here are the precent changes in the level:

   3.5, 8.1, 7.4, 4.0, 0.7, 4.9, 8.4, 7.0, 5.5.

   (Note: $\bar{x} = 5.5$ and $\sum_{i=1}^{9}(x_i - \bar{x})^2 = 50.68$.) For this problem suppose that the data comes from a normal distribution with mean μ and variance σ^2. Give a 90% confidence interval for μ when:
   (a) it is assumed $\sigma^2 = 6$
   (b) σ^2 is unknown

3. Here are data from a study of seat belt use for Hispanic and white male drivers in Chicago:

Group	Drivers	Belted
Hispanic	539	286
White	292	164

   Is there evidence that the two groups use seat belts at a different rate? Check this hypothesis at the $\alpha = 0.05$ level.

4. For this problem suppose $X_1, X_2, \ldots, X_n \overset{\text{iid}}{\sim} \Gamma(3, \lambda)$. You may use the fact that:

$$\int x^2 e^{-x}\, dx = -\left(2 + 2\,x + x^2\right) \mathrm{e}^{-x} + C$$

   (a) Find the Maximum Likelihood Estimator for λ.
   (b) Find the Method of Moments Estimator for λ.

5. Suppose you collect the following data where x_i is the explanatory variable and Y_i is the response variable. You model this situation with the simple linear model $Y_i = \beta_1 x_i + \beta_0 + \epsilon_i$, where $\epsilon_i \overset{\text{iid}}{\sim} N(0, \sigma^2)$.

x_i	Y_i
1	2
1	0
-1	0
-1	-2
0	0

(Note: The data was created entirely for the purpose of easy calculation and not to be at all realistic.)

(a) Suppose you want to test the null hypothesis that $\beta_1 = 0$. Compute the T statistic for the test and say what distribution it would have under the null. Please give the exact value of the T statistic rather than a decimal approximation.

(b) Test the hypothesis that $\sigma^2 = 1$ versus the alternative that $\sigma^2 > 1$ at the 0.05-level.

6. Suppose that for any x_i, Y_i is observed where $Y_i = 2x_i + 1 + \epsilon_i$, and $\epsilon_i \overset{\text{iid}}{\sim} N(0, 4)$. Suppose that $n = 26$ data points are collected where the x_i's are chosen so that $\sum_{i=1}^{26}(x_i - \bar{x})^2 = 4$.

(a) Find $P(\hat{\beta}_1 > 3 \text{ and } S^2 > 5.53)$, where $\hat{\beta}_1$ and S are the usual estimators for fitting a simple linear regression.

(b) Determine if $\hat{\beta}_1 S^2$ an unbiased estimator of $\beta_1 \sigma^2$. (make sure to explain).

(c) Is $\hat{\beta}_1 S$ an unbiased estimator of $\beta_1 \sigma$? (make sure to explain).