For these problems let $\mathbb{A}$ be the set of algebraic numbers.

1. Show $\mathbb{Q} \subseteq \mathbb{A}$.
2. Show $\mathbb{Q} \neq \mathbb{A}$.
3. Show $x \in \mathbb{A}$ if and only if there exists a polynomial, $p(x)$ with **integer** coefficients such that x is a root of $p(x)$.

For the following two problems use this result, known as the division algorithm for polynomials:

Theorem 1 *If $p_1(x), p_2(x)$ are (non-zero) polynomials then there exists unique polynomial $q(x), r(x)$ such that:*

$$p_1(x) = p_2(x)q(x) + r(x)$$

where deg $r(x) <$ deg $p_2(x)$.

4. Find the $q(x)$ and $r(x)$ from above for the following two examples:
   (a) $p_1(x) = x^3 - 2x + 1$ and $p_2(x) = x^2 - 1$.
   (b) $p_1(x) = x^5 + 15x^3 + 3x$ and $p_2(x) = x^{12} + x^8 - x^3 + 1$.
5. Show that if $p(x)$ is a polynomial then a is a root of $p(x)$ if and only if $x - a$ divides $p(x)$.