

1. Suppose that R is the radius of convergence of the power series: $\sum_{k=0}^{\infty} a_k(x - x_0)^k$. Show that:

(a) The power series converges absolutely for all x such that $|x - x_0| < R$

(b) The power series diverges for all x such that $|x - x_0| > R$

(c) If $0 < r < R$ then $f_n(x) = \sum_{k=0}^n a_k(x - x_0)^k$ converges uniformly to $f(x) = \sum_{k=0}^{\infty} a_k(x - x_0)^k$ on $[x_0 - r, x_0 + r]$.

(d) $f(x)$ defined above is continuous on $(x_0 - R, x_0 + R)$.