1. Consider the power series $\sum_{k=0}^{\infty} a_k(x-x_0)^k$.

   (a) Show that radius of convergence of the series is ∞ if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 0$.

   (b) Show that radius of convergence of the series is 0 if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$.

2. Consider the power series $\sum_{k=0}^{\infty} a_k(x-x_0)^k$.

   (a) Show that radius of convergence of the series is ∞ if $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = 0$.

   (b) Show that radius of convergence of the series is 0 if $\limsup_{n\to\infty} \sqrt[n]{|a_n|} = \infty$.

3. Suppose $\{a_n\}$ is defined by:

$$a_n = \begin{cases} n^{-n} & \text{if } n \text{ is odd} \\ 2^{-n} & \text{if } n \text{ is even} \end{cases}$$

   Find the radius of convergence of $\sum_{k=0}^{\infty} a_k(x-3)^k$