Suppose V is a vector space over $\mathbb{R}$ and suppose that for all $x, y \in V$ there is a real number called the inner product of x, y denoted $< x, y >$ satisfying the following properties:

1. $< x, y > = < y, x >$ for all $x, y \in V$
2. $< cx_1 + x_2, y > = c < x_1, y > + < x_2, y >$ for all $x_1, x_2, y \in V$ and $c \in \mathbb{R}$.
3. $< x, x > \geq 0$ for all $x \in V$ with $< x, x > = 0$ if and only if $x = \vec{0}$.

1. Show the following known as the Cauchy-Schwarz Theorem:

$$< x, y >^2 \leq < x, x > < y, y >$$

   Hint we proved it in class for $\mathbb{R}^n$ and the proof is basically the same.

2. Show that $||x|| = \sqrt{< x, x >}$ is a norm on V. (Hint: Use Cauchy-Schwarz).