1. Suppose that d_1, d_2 are two metrics on X and there exists $0 < c_1 \leq c_2$ such that for all $x, y \in X$, $c_1 d_1(x,y) \leq d_2(x,y) \leq c_2 d_1(x,y)$. Show:

   (a) A set U is open with respect to d_1 if and only if it is open with respect to d_2.

   (b) A sequence x_n converges to x with respect to d_1 if and only if x_n converges to x with respect to d_2.

   Note d_1 and d_2 are said to be equivalent metrics.

2. Consider the vector space $\mathbb{R}^n$.

   (a) Show that for all $v \in \mathbb{R}^n$, $||v||_u \leq ||v||_1 \leq n\,||v||_u$ and $||v||_u \leq ||v||_2 \leq \sqrt{n}\,||v||_u$

   (b) Show that the metrics corresponding $||\cdot||_1$,$||\cdot||_2$ and $||\cdot||_u$ are equivalent.

3. Consider the vector space $c([a,b])$.

   (a) Show that for all $f \in c([a,b])$, $||f||_1 \leq (b-a)\,||f||_u$.

   (b) Suppose $a = 0$ and $b = 1$. Show by examples that it is possible for $||f||_1 = 1$ but $||f||_u$ to be arbitrarily large. (i.e. for all $N \in \mathbb{N}$ there exists $f \in c([0,1])$ such that $||f||_1 = 1$ but $||f||_u \geq N$.)

4. For this problem you can use what you know about trigonometric functions from calculus even stuff we have not proved. (But you could prove them: for example to show that $\cos(\pi/2) = 2\cos^2(\pi/4) - 1$ and thus $\cos(\pi/4) = \pm 1/\sqrt{2}$ and then using the fact that $\cos(x)$ is positive on $(0, \pi/2)$ to show $\cos(\pi/4) = 1/\sqrt{2}$. Similarly for $\sin(\pi/4)$. The point is you don't have to prove these things. But I digress. Anyway what were we talking about. Oh yeah) Find $d(\sin(x), \cos(x))$ using the metrics $||\cdot||_1$,$||\cdot||_2$, and $||\cdot||_u$ on $c([0,\pi])$.