

For each assume (X, d) is a metric space.

1. Suppose $\{x_n\}$ is a sequence in X and $x_n \rightarrow x$ and $f : X \rightarrow Y$ is continuous function to a metric space Y . Prove $f(x_n) \rightarrow f(x)$.
2. Suppose $A \subseteq X$.
 - (a) Prove $x \in X$ is an accumulation point of A if and only if there exists a sequence $\{a_n\} \subseteq A$, with $a_n \neq x$ for all n such that $a_n \rightarrow x$.
 - (b) Show A is closed in X if and only if for all sequences $\{a_n\} \subseteq A$ such that $a_n \rightarrow a \in X$ then $a \in A$.
3. Show if X is compact then for all $x \in X$ there exists and $r > 0$ such that $B_r(x) = X$. This is what it means for X to be bounded.