

1. Give the definition of a set $U \subseteq \mathbb{R}$ being an **open** set.
2. State the Intermediate Value Theorem (technically the Bolzano Intermediate Value Theorem).
3. State the definition of an interval (not necessarily of finite length).
4. Show if I is an interval and $f : I \rightarrow \mathbb{R}$ is continuous then $f(I)$ is an interval.
5. Use the Bolzano-Weierstrass Theorem to show that if $f : K \rightarrow \mathbb{R}$ is a continuous function where $K \subseteq \mathbb{R}$ is compact then $f(K)$ is compact. (Hint: Remember that $f : D \rightarrow \mathbb{R}$ is continuous at $c \in D$ if and only if for every sequence $\{c_n\} \subseteq D$ that converges to c , $\{f(c_n)\}$ converges to $f(c)$).
6. Show if $a, b \in \mathbb{R}$ with $a < b$ and $f : [a, b] \rightarrow \mathbb{R}$ continuous then there exists $m, M \in \mathbb{R}$ with $m \leq M$ (note m could equal M so the “interval” would contain one element) such that $f([a, b]) = [m, M]$.