1. Finish the proof that if $\alpha, \beta, \gamma \in [a, b]$ and $f \in \mathcal{R}[a, b]$, then:

$$\int_\alpha^\gamma f = \int_\alpha^\beta f + \int_\beta^\gamma f$$

2. Suppose that $f \in \mathcal{R}[a, b]$ and $a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$. Then show that $f \in \mathcal{R}[x_{i-1}, x_i]$ for all $1 \leq i \leq n$ and:

$$\int_a^b f = \sum_{i=1}^{n} \int_{x_{i-1}}^{x_i} f.$$

3. Suppose $f, F : [a, b] \to \mathbb{R}$ with $F$ differentiable and $F'(x) = f(x)$ for all $x \in [a, b]$ and $f \in \mathcal{R}[a, b]$. Suppose $\alpha, \beta \in [a, b]$ show:

$$\int_\alpha^\beta f = F(\beta) - F(\alpha)$$

4. Let $F : [0, 3] \to \mathbb{R}$ defined by:

$$F(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq 1 \\ x - 1 & \text{if } 1 \leq x \leq 2 \\ 1 & \text{if } 2 \leq x \leq 3. \end{cases}$$

   (a) Prove that $F \in c([0, 3])$

   (b) Find an $f : [0, 3] \to \mathbb{R}$ such that $F'(x) = f(x)$ for all but a finite set.

   (c) Find $\int_0^3 f$.

   (d) Find $\int_0^3 F$.