1. Last time I said you may assume that for all $x \in \mathbb{R}$, $\lim_{x\to\infty} \frac{x^n}{n!} = 0$. Now you will prove it. Fix $x \in \mathbb{R}$ and let $a_n = \frac{|x|^n}{n!}$. Do the following:

   (a) Show that a_n is eventually decreasing. (Hint: look at the ratio of successive terms)

   (b) Prove that a_n converges. (Hint: Use a theorem from 360)

   (c) Prove $\lim_{n\to\infty} a_n = 0$. (Hint: Use limit laws)

   (d) Finally argue that $\lim_{x\to\infty} \frac{x^n}{n!} = 0$.

2. For this problem you may use any of the results in the previous homework (whether you proved it or not). Remember for $a > 0$ and $x \in \mathbb{R}$, we defined $a^x = \exp(x \ln(a))$.

   (a) Show that if $a > 0$ and $x, y \in \mathbb{R}$ show that $a^{x+y} = a^x a^y$.

   (b) Show that if $a > 0$ and $x \in \mathbb{R}$ then $\ln(a^x) = x \ln(a)$.

   (c) Show that if $a > 0$ and $x, y \in \mathbb{R}$ show that $a^{xy} = (a^x)^y$.

   (d) Let $a \in \mathbb{R}$ and define $f(x) = x^a$. Show that f is differentiable on $(0, \infty)$, and find $f'(x)$.

3. For each $n \in \mathbb{N}$ let $f_n(x) = \sum_{k=0}^{n} \frac{x^k}{k!}$. Show that for $M > 0$, $f_n \to \exp$ uniformly on $[-M, M]$.