1. Consider the interval $[0,6]$ and the function $f(x)=x^3$. For each of the following tagged partitions find:

   i. $\left|\left|\dot{\mathcal{P}}\right|\right|$

   ii. $\mathrm{S}(f,\dot{\mathcal{P}})$

   (a) $\dot{\mathcal{P}} = \left((0,2,6)\right),(0,5))$

   (b) $\dot{\mathcal{P}} = \left((0,1,2,5,6)\right),(0.5,1.5,3.5,5.5))$

   (c) $\dot{\mathcal{P}} = \left((0,1,2,3,6)\right),(1,1,3,3))$

2. Suppose $f \in \mathcal{R}[a,b]$ and $\dot{\mathcal{P}}_n$ be any sequence of tagged partitions such that $\lim\limits_{n\to\infty}\left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$. Show that $\lim\limits_{n\to\infty} S(f,\dot{\mathcal{P}}_n) = \displaystyle\int_a^b f$.

3. Prove by induction that for all $n \in \mathbb{N}$, $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. (I know this is a math logic question but do it for old times' sake.)

4. Let $f(x) = x^2$. Assume that $f \in \mathcal{R}[0,1]$ (we will soon show that all continuous functions are Riemann integrable). Use the previous two problems to show that $\int_0^1 x^2 = \frac{1}{3}$.

5. Suppose $f \in b([a,b])$ and the there exists two sequences of tagged partitions such that $\lim\limits_{n\to\infty}\left|\left|\dot{\mathcal{P}}_n\right|\right| = 0$ and $\lim\limits_{n\to\infty}\left|\left|\dot{\mathcal{Q}}_n\right|\right| = 0$, but $\lim\limits_{n\to\infty} S(f,\dot{\mathcal{P}}_n) \neq \lim\limits_{n\to\infty} S(f,\dot{\mathcal{Q}}_n)$. Show $f \notin \mathcal{R}[a,b]$.

6. Consider the function:

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

   Prove $f \notin \mathcal{R}[0,1]$.