

1. Show that if  $\{f_n\}$  converges to  $f$  uniformly on  $A \subseteq \mathbb{R}$  then it converges pointwise on  $A$ .
2. Show that  $\{f_n\}$  converges to some  $f$  uniformly on  $A \subseteq \mathbb{R}$  then  $\{f_n\}$  is Cauchy with respect to the uniform norm.
3. Show that if  $f(x)$  is differentiable on  $\mathbb{R}$  and for all  $x \in \mathbb{R}$ ,  $f'(x) = f(x)$  and  $f(0) = 1$  then  $f(x) = \exp(x)$ .
4. Let  $0 < L < 1$  and  $f_n(x) = \sum_{k=0}^n x^k$ . Show that  $f_n$  converges uniformly to some  $f$  on  $[-L, L]$ .

5. Show:

$$\ln(2) = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{k \cdot 2^k}$$

6. Show that  $\ln(2) > \frac{2}{3}$ .