1. Suppose that function $\{S_n\}$ and $\{C_n\}$ are a sequence of functions, such that $c_0 = 1$, for each $n \in \mathbb{N}$, $S_n(x) = \int_0^x C_n$ and $C_{n+1}(x) = 1 - \int_0^x S_n$. Prove by induction (yes show your induction) that for all $n \in \mathbb{N}$:

   (a) $S_n(0) = 0$

   (b) $C_n(0) = 1$

   (c) $S_n(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$

   (d) $C_n(x) = \sum_{k=0}^{n} \frac{(-1)^k x^{2k}}{(2k)!}$

   Hint: Just do one induction to do all of these.

2. Remember that a set is said to be complete with respect to the uniform norm is every Cauchy sequence in the set converges in the set (i.e. the limit with respect to the uniform norm is also in the set.) Show that if $M > 0$ then $c[-M, M]$ (the set of continuous functions whose domain is $[-M, M]$) is complete with respect to the uniform norm.

3. Let $A \subseteq \mathbb{R}$ and define $p(A)$ be the set of all polynomial functions whose domain is A. Also let $M > 0$.

   (a) Show $\exp(x) \notin p([-M, M])$. Hint: Find some property that all polynomials have the $\exp(x)$ does not have.

   (b) Show that $p([-M, M])$ is not complete with respect to the uniform norm.