

1. Consider the power series: $\sum_{k=1}^{\infty} kx^k$.

(a) Find its radius of convergence

(b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

(c) Find the sum of $\sum_{k=1}^{\infty} \frac{k}{2^k}$.

2. Consider the power series: $\sum_{k=1}^{\infty} k^2 x^k$.

(a) Find its radius of convergence

(b) In the interior of the interval of convergence, give a closed form expression (one without summation) for what the function the series converges to.

(c) Find the sum of $\sum_{k=1}^{\infty} \frac{k^2}{3^k}$.

3. Let X be a nonempty set and define:

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y. \end{cases}$$

Show (X, d) is a metric space.