1. Show that if $(V, ||\cdot||)$ is a normed vector space then it is a metric space under $d(v_1, v_2) = ||v_1 - v_2||$.

2. Show that $||\cdot||_1$ is a norm on $\mathbb{R}^n$ for all n .

3. Suppose $\vec{x} \in \mathbb{R}^n$ show:

   (a) $||\vec{x}||_u \leq ||\vec{x}||_1 \leq n||\vec{x}||_u$

   (b) $||\vec{x}||_u \leq ||\vec{x}||_2 \leq \sqrt{n}||\vec{x}||_u$

4. Let $a \in \mathbb{R}^\infty$ with $\mathbf{a} = \{a_n\}_{n=1}^\infty$

$$a_n = \begin{cases} 5^{-n} & \text{if } n \text{ is odd} \\ 2^{-n} & \text{if } n \text{ is even} \end{cases}$$

   Find:

   (a) $||\mathbf{a}||_1$

   (b) $||\mathbf{a}||_2$

   (c) $||\mathbf{a}||_u$

   Hint: If a series converges absolutely then the series can be summed in any order.