

1. Show $\cos'(x) = -\sin(x)$
2. Remember now that \sin and \cos are the same as what we called S and C so you can use all the properties we showed for S and C . Define $\tan(x) = \frac{\sin(x)}{\cos(x)}$.

(a) Show $\tan(x) : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow \mathbb{R}$ is an increasing, continuous, bijection.

(b) Define $\arctan(x) : \mathbb{R} \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$ to be the functional inverse of $\tan(x)$. Show it is an increasing, continuous, bijection.

(c) Find $\arctan'(x)$.

Note for topology students. Notice that these are the functions we needed (but didn't have) in topology class to show that for all $a < b$, (a, b) is homeomorphic to \mathbb{R} with both having the usual topology. So your topology class is now complete.

3. Show if $\sum_{k=1}^{\infty} a_k$ converges then $\lim_{k \rightarrow \infty} a_k = 0$.

4. Suppose for some $M > 0$, $0 \leq Mb_k \leq a_k$ eventually. Show that if $\sum b_k$ diverges, then $\sum a_k$ diverges.

5. Suppose that $\left| \frac{a_{k+1}}{a_k} \right| > 1$ eventually. Show that $\sum a_k$ diverges.