

1. Consider  $\vec{x} = (1, -2, 0, 1)$  and  $\vec{y} = (2, 0, -3, 1)$  to be elements of the inner product space on  $\mathbb{R}^4$ .
  - (a) Find  $|\langle \vec{x}, \vec{y} \rangle|$ .
  - (b) What upper bound does Cauchy-Schwarz give for this quantity?
2. Consider  $f(x) = x$  and  $g(x) = \exp(x)$  to be elements of the inner product space  $L^2([0, 1])$ .
  - (a) Find  $|\langle f, g \rangle|$ .
  - (b) What upper bound does Cauchy-Schwarz give for this quantity?
3. Suppose  $X$  is an inner product space and fix  $y \in X$ . Define a function  $f : X \rightarrow \mathbb{R}$  by  $f(x) = \langle x, y \rangle$ . Show  $f$  is continuous (using the metric space induced by inner product on  $X$  and the usual metric space on  $\mathbb{R}$ .) Hint: Use Cauchy-Schwarz but you knew that already.
4. Show that  $\mathbb{R}^n$  is complete under  $\|\vec{x}\|_2$ , and hence  $\mathbb{R}^n$  is a Hilbert Space under the usual dot product.