1. Let A be an $m \times n$ matrix. Thus we can think of $A \in L(\mathbb{R}^n, \mathbb{R}^m)$. Let $\mathbb{R}^m$ (the co-domain) have the uniform norm, $||\cdot||_u$. Find (and describe) the operator norm of A, $||A||_{\mathrm{OP}}$ where:

   (a) $\mathbb{R}^n$ (the domain) has $||\cdot||_u$.

   (b) $\mathbb{R}^n$ (the domain) has $||\cdot||_1$.

2. For each of the 4 norms given for A above and in class find $||A||$ for:

$$A = \begin{bmatrix} 1 & -4 & 0 \\ -2 & 3 & 0 \\ -3 & 5 & 8 \\ -4 & 1 & -2 \end{bmatrix}$$

3. Let X be a vector space and $||\cdot||_a$ and $||\cdot||_b$. Suppose for all $x \in X$, $||x||_a \leq ||x||_b$. There are 4 possible norms on $L(X, Y)$, call them $||\cdot||_{aa}$, $||\cdot||_{ab}$, $||\cdot||_{ba}$, and $||\cdot||_{bb}$ where for example $||\cdot||_{ab}$ is the operator norm with a norm on the domain and b norm on the co-domain. As best as possible find the order relationship between the four norms (Hint: two will be incomparable).

4. Check that the last question works on the 4 norms we had for $m \times n$ matrices.