1. Suppose $\{a_n\}$ is a sequence. Show:

   (a) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \beta < 1$ then $\sum_{k=0}^{\infty} a_k$ converges absolutely.

   (b) If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \beta > 1$ then $\sum_{k=0}^{\infty} a_k$ diverges.

   (c) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \beta < 1$ then $\sum_{k=0}^{\infty} a_k$ converges absolutely.

   (d) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \beta > 1$ then $\sum_{k=0}^{\infty} a_k$ diverges.

2. Let a_n defined as:

$$a_n = \begin{cases} \frac{1}{2^{(n+1)/2}.3^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \frac{1}{2^{n/2}.3^{n/2}} & \text{if } n \text{ is even} \end{cases}$$

   Show that $\sum_{k=0}^{\infty} a_k$ converges.

3. Let a_n defined as:

$$a_n = \begin{cases} \frac{1}{2^{(n+1)/2}} & \text{if } n \text{ is odd} \\ \frac{1}{3^{n/2}} & \text{if } n \text{ is even} \end{cases}$$

   Show that $\sum_{k=0}^{\infty} a_k$ converges.

4. Find all x (except possibly 2 points) such that the following series converge:

   (a) $\sum_{k=0}^{\infty} \frac{(x+2)^k}{5^k}$

   (b) $\sum_{k=1}^{\infty} \frac{(x-1)^k}{k^2 3^k}$

   (c) $\sum_{k=0}^{\infty} \frac{(2x)^k}{k!}$

   (d) $\sum_{k=1}^{\infty} \left(1+\frac{x}{k}\right)^k$

   (That is you can say “I don’t know” for at most 2 points per problem, but you must say what all the other x do).